

On black hole thermodynamics and the entropy function formalism

Xian-Hui Ge and Fu-Wen Shu*

Asia Pacific Center for Theoretical Physics, Pohang 790-784, Korea

E-mail: gexh@apctp.org, fwshu@apctp.org

ABSTRACT: From the black hole thermodynamics point of view, we show that the entropy function \mathbf{f} and the free energy F are related via $\mathbf{f} = e_I q_I + \Omega_H q_I A_\phi^{I'} - \frac{\partial F}{\partial r} |_{r_H}$. Assuming the entropy function is known for extremal black holes, we propose an approach to calculate the entropy of non-extremal cases by slightly moving the extremal black hole geometry from extremality. The entropy of non-extremal $D1D5$ - and $D1D5p$ -branes in the presence of higher derivative corrections are computed as concrete examples. An attempt has also been made to explain why the entropy function method can calculate the corrected entropy without knowing the exact form of black hole solution in higher derivative gravity theories.

KEYWORDS: Black Holes, Black Holes in String Theory.

*XHG and FWS contribute to this paper equivalently.

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1. Introduction

The entropy function formalism proposed by Sen, turns out to be a very powerful method in computing extremal black hole entropy in the presence of higher derivative gravity terms [1]. Usually, the entropy function method is composed of three steps. The first is to write near horizon geometry of an extremal black hole in n dimensions into $AdS_2 \times S^{n-2}$ with constant radii v_i . The next step is to assume the coupled electric, magnetic fields, and scalar fields to be some constants u_i . Finally, introducing a function (a function of v_i and u_i), which is the integral of Lagrangian density over the horizon S^{n-2} and performing the Legendre transformation of this function with respect to electric field strengths and extremizing it with respect to the scalars v_i and u_i . The entropy function method extends the understanding of the attractor mechanism by showing that not only scalar fields but also every parameter of the near horizon geometry of an extremal black hole can be fixed by extremizing a function evaluated near the horizon.

Actually, the entropy function formalism makes the computation of Wald's entropy formula [2], a more general formalism for computing black hole entropy in the presence of higher derivative terms, much simpler. Even though the entropy function formalism is initially claimed to work for supersymmetric extremal black holes in supergravity theories, there are several attempts to apply the entropy function to non-supersymmetric extremal black holes [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and even non-extremal black holes [16, 17]. However, several problems are required to be clarified on the entropy function formalism:

1). It is amazing that only from the near horizon geometry, the entropy function formalism computes the entropy of an extremal black hole by means of a Legendre transformation. However, people usually use the full spacetime geometry to compute the black hole entropy. Therefore, how to relate the entropy function formalism with the traditional understanding of black hole thermodynamics? Fortunately, this problem has been discussed by several authors in Ref[18, 19]. The authors established the connection between the entropy function method and the traditional Euclidean approach for black hole thermodynamics at zero temperature limit and found that the entropy function agrees with the zero temperature limit of the Euclidean action.

2). In some of the literature, the entropy function method has been extended to non-extremal black holes. However, there are no attractor mechanisms for non-extremal solutions. How to grantee that the extension of the entropy function formalism is safe? This is the main purpose of this paper. We first discuss the relations between black hole thermodynamics and the entropy function formalism and then we propose a method to compute the entropy of near extremal black holes when higher derivative gravity terms are taken into account. The main idea of our method is that we slightly move the black hole geometry from extremality where the extremal entropy function is still valid. In this case, the near-extremal black hole entropy and the extremal black hole entropy function approximately satisfy a equation $TS' + T'S \approx \mathbf{f}_{ex}$. One can solve this equation to obtain the near-extremal black hole entropy.

3). The near horizon metric $ds^2 = v_1(-r^2 dt^2 + \frac{dr^2}{r^2}) + v_2 d\Omega_{n-2}^2$ was used in obtaining higher

derivative corrections to extremal black hole entropy. It seems that one can calculate the corrected entropy without knowing the exact metric of extremal black holes in higher derivative gravity theory. We will explain this point by showing that the $AdS_2 \times S^{n-2}$ geometry does not change in the presence of curvature square terms.

It is found in the present paper that the entropy function \mathbf{f} and the free energy F are related by $\mathbf{f} = e_I q_I + \Omega_H q_I A_\phi^{I'} - \frac{\partial F}{\partial r} |_{r_H}$. Our results imply that applying the entropy function method to non-extremal black holes might have difficulties, because the near horizon geometry of non-extremal solutions in higher derivative gravity theory are not always $AdS_2 \times S^{n-2}$. As to the third point, Sen speculated that *in any general covariant theory of gravity coupled to matter fields, the near horizon geometry of a spherically symmetric extremal black hole in n dimensions has $SO(2,1) \times SO(D-1)$ isometry*[20]. In four and five dimensions, this postulate was proved in [21, 22, 23]. The near horizon geometry of non-extremal black holes in the presence of higher derivative gravity terms might not have the $SO(2,1) \times SO(D-1)$ isometry, and also there is no attractor mechanism for non-extremal black holes. However, in order to overcome the above difficulties, we focus on a class of black holes which have extremal correspondences and whose entropy can be obtained from the entropy function formalism. Once the behavior of an extremal black hole is known by using the entropy function formalism, we put some “excess” energy into the extremal black hole and make it “near-extremal” with low but non-zero temperature. With this non-strict approach, we can calculate the entropy of non-extremal black holes including the higher derivative corrections.

The paper is constructed as follows. In section 2 we establish the connections between black hole thermodynamics and the entropy function formalism for non-extremal black holes. In section 3 we review the relations between the near horizon geometry of extremal black holes and the higher derivative gravity corrections. In section 4 and section 5, we give two examples of how to extend the entropy function to non-extremal black holes when higher derivative corrections are taken into account. Section 6 contains concluding remarks.

2. Entropy function and Euclidean black hole thermodynamics

To be general, we start from an n -dimensional universal metric

$$ds^2 = g_{ij} dx^i dx^j. \quad (2.1)$$

According to Sen’s entropy function method, the metric is required to deform into a metric of extremal black holes with near horizon geometry $AdS_2 \times S^2$, namely [1]

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 d\Omega_{n-2}^2. \quad (2.2)$$

where v_1 and v_2 are assumed to be constants. We do not deform the metric (2.1) into near horizon geometry and the advantage of the deformed metric (2.2) will be discussed later. Now we denote f as function of the lagrangian density $\sqrt{-\det g} \mathcal{L}$ on the horizon with the

following form ¹

$$f = \int dx^3 \dots dx^n \sqrt{-\det g} \mathcal{L}, \quad (2.3)$$

where the lagrangian density may include gravitational fields, scalar fields, gauge fields and covariant derivatives of these fields i.e. $\mathcal{L} = \mathcal{L}^{tree} + \mathcal{L}^{corr}$, and $\{x^3 \dots x^n\}$ are the angular coordinates. The Euclidean action can be obtained by doing a Wick rotation $t \rightarrow i\tau$, i.e.²

$$I_E = \int d\tau dr f. \quad (2.4)$$

For a stationary black hole, we have

$$I_E = \beta \int dr f = \beta F, \quad (2.5)$$

where β is the Euclidean time and F is the *free energy* according to Hawking and Page[24].

Now we would like to discuss the relationship between the function f and Wald entropy formula. Consider the lagrangian as an n -form $\mathbf{L}(\psi)$, where $\psi = \{g_{ab}, R_{abcd}, \Phi_s, F_{ab}^I\}$ denotes the dynamical fields considered in this paper, including the spacetime metric g_{ab} , the corresponding Riemann tensor R_{abcd} , the scalar fields $\{\Phi_s, s = 0, 1, \dots\}$, and the $U(1)$ gauge fields $F_{ab}^I = \partial_a A_b^I - \partial_b A_a^I$ with the corresponding potentials $\{A_a^I, I = 1, \dots\}$. Under this definition, the variation of \mathbf{L} is

$$\delta \mathbf{L} = \mathbf{E}_\psi \delta \psi + d\mathbf{\Theta}, \quad (2.6)$$

where $\mathbf{\Theta}$ is an $(n-1)$ -form, which is called *symplectic potential form*, \mathbf{E}_ψ corresponds to the equations of motion for the metric and other fields. Let ξ be any smooth vector field on the space-time manifold, then one can define a *Noether current form* as

$$\mathbf{J}[\xi] = \mathbf{\Theta}(\psi, \mathcal{L}_\xi \psi) - \xi \cdot \mathbf{L}. \quad (2.7)$$

We will consider ξ to be a killing vector vanishing on the bifurcation horizon. Thus, $\mathcal{L}_\xi \psi = 0$ and $\mathbf{\Theta}(\psi, \mathcal{L}_\xi \psi) = 0$. The fact that $d\mathbf{J}[\xi] = 0$ will be preserved when the equations of motion are satisfied shows that a locally constructed $(n-2)$ -form $\mathbf{Q}[\xi]$ can be introduced and an “on shell” formula can be obtained

$$\mathbf{J}[\xi] = d\mathbf{Q}[\xi]. \quad (2.8)$$

Wald’s analysis based on the first law of black hole thermodynamics showed that for general stationary black holes, the black hole entropy is a kind of Noether charge at horizon [2] and can be expressed as

$$S_{BH} = 2\pi \int_{\mathcal{H}} \mathbf{Q}[\xi], \quad (2.9)$$

¹The function defined here is different from that of Sen’s definition in that $f = 2\pi T' f_{sen}$.

²More precisely, by doing a Wick rotation $t \rightarrow i\tau$, function f should be replaced by

$$f_E \equiv \int dx^3 \dots dx^n \sqrt{\det g_E} \mathcal{L}(i\tau).$$

However, we do not need to distinguish f from f_E since they have the same value.

where ξ represents the Killing field on the horizon, and \mathcal{H} is the bifurcation surface of the horizon. It should be noted that the Killing vector field has been normalized to have unit surface gravity. Integrating over a Cauchy surface \mathcal{C} on Eq.(2.8) and using Eq.(2.7), we find that

$$\int_{\mathcal{C}} \mathbf{J} = - \int_{\mathcal{C}} \xi \cdot \mathbf{L} = \int_{\mathcal{C}} d\mathbf{Q}[\xi] \quad (2.10)$$

In an asymptotically flat spacetime, we have the interior boundary at the horizon \mathcal{H} and the outer boundary at infinity ∞ . We obtain

$$\int_{\mathcal{C}} d\mathbf{Q}[\xi] = \int_{\infty} \mathbf{Q} - \int_{\mathcal{H}} \mathbf{Q}, \quad (2.11)$$

where we have used the Stokes theorem. The Euclidean action I_E corresponds to [2]

$$I_E = -\frac{1}{T} \left[\int_{\mathcal{C}} \xi \cdot \mathbf{L} + \int_{\infty} t \cdot \mathbf{B} \right], \quad (2.12)$$

where \mathbf{B} is an $(n-1)$ -form defined as

$$\delta \int_{\infty} t \cdot \mathbf{B} = \int_{\infty} t \cdot \boldsymbol{\Theta}.$$

From Eqs.(2.5), (2.10), (2.11) and (2.12), one finds that

$$F + \int_{\infty} t \cdot \mathbf{B} = \int_{\infty} \mathbf{Q} - \int_{\mathcal{H}} \mathbf{Q} \quad (2.13)$$

Note that the “canonical energy” \mathcal{E} and the “canonical angular momentum” \mathcal{J} are defined by [2],

$$\mathcal{E} = \int_{\infty} (\mathbf{Q}[t] - t \cdot \mathbf{B}), \quad (2.14)$$

$$\mathcal{J} = - \int_{\infty} \mathbf{Q}[\varphi]. \quad (2.15)$$

For an asymptotically flat black hole metric, one can choose the Killing vector as $\xi^a = t^a + \Omega_H^\mu \varphi_{(\mu)}^a$ with Ω_H the angular velocity of the horizon, then one obtains

$$F = \mathcal{E} - \Omega_H^\mu \mathcal{J}_\mu - \int_{\mathcal{H}} \mathbf{Q}[\xi^a] \quad (2.16)$$

The variation of Eq.(2.16) leads to

$$\delta F = \delta \mathcal{E} - \Omega_H^\mu \delta \mathcal{J}_\mu - \delta \int_{\mathcal{H}} \mathbf{Q}[\xi^a] \quad (2.17)$$

The Noether charge is composed of the contributions from the $U(1)$ gauge fields and gravitational fields, that is to say

$$\mathbf{Q} = \mathbf{Q}^F + \mathbf{Q}^g + \dots \quad (2.18)$$

where

$$\mathbf{Q}_{a_1 \dots a_{n-2}}^F = \frac{\partial \mathcal{L}}{\partial F_{ab}^I} \xi^c A_c^I \epsilon_{aba_1 \dots a_{n-2}}, \quad (2.19)$$

$$\mathbf{Q}_{a_1 \dots a_{n-2}}^g = -\frac{\partial \mathcal{L}}{\partial R_{abcd}} \nabla_{[c} \xi_{d]} \epsilon_{aba_1 \dots a_{n-2}}. \quad (2.20)$$

Now, following the work of Ref.[25], we consider a stretched region near the horizon ranged from r_H to $r_H + \delta r$, i.e.

$$\delta \int_{\mathcal{H}} \mathbf{Q}[\xi^a] = \int_{r_H}^{r_H + \delta r} (\mathbf{Q}^F[\xi^a] + \mathbf{Q}^g[\xi^a]) \quad (2.21)$$

Taking account of the Killing equation, we have $\nabla_{[a} \xi_{b]} = 2\kappa \epsilon_{ab}$ (where κ is the surface gravity of the hole), and the two parts are found to be [25]

$$\int_{r_H}^{r_H + \delta r} \mathbf{Q}^g[\partial_t + \Omega_H \partial_\phi] = \delta r [\kappa' E + \kappa E']_{r_H} + \mathcal{O}(\delta r^2), \quad (2.22)$$

$$\int_{r_H}^{r_H + \delta r} \mathbf{Q}^F[\partial_t + \Omega_H \partial_\phi] = -(q_I e_I + \Omega_H q_I A'_\phi) \delta r + \mathcal{O}(\delta r^2). \quad (2.23)$$

where $E(r)$ is defined as

$$E(r) \equiv - \int_{\mathcal{H}} \frac{\partial \mathcal{L}}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} dx^1 \dots dx^{n-2}. \quad (2.24)$$

The above formula is exactly the Wald formula for entropy without the factor 2π [2]. According to the definition of entropy function in (4.4) of [25], we find $E(r)$ is related to the entropy by $2\pi E(r_H) = S$. e_I and the $U(1)$ electrical-like charges in Eq. (2.23) are defined to be

$$e_I \equiv F_{rt}^I(r_H), \quad (2.25)$$

$$A_\phi'^I = (\partial_\phi)^a A_a^I, \quad (2.26)$$

$$\begin{aligned} q_I &\equiv - \int_r \frac{1}{(n-2)!} \frac{\partial \mathcal{L}}{\partial F_{ab}^I} \epsilon_{aba_1 \dots a_{n-2}} dx^{a_1} \wedge \dots \wedge dx^{a_{n-2}} \\ &= - \frac{\partial}{\partial e_I} \int_{r_H} \frac{\mathcal{L}}{2(n-2)!} \epsilon^{ab} \epsilon_{aba_1 \dots a_{n-2}} dx^{a_1} \wedge \dots \wedge dx^{a_{n-2}} = \frac{\partial f(r_H)}{\partial e_I}, \end{aligned} \quad (2.27)$$

where we have written $F_{ab}^I(r_H)$ as $-e_I \epsilon_{ab}$. If the near horizon extension $r_H \rightarrow r_H + \delta r$ is also done for the free energy, we find that

$$\delta F = - \int_{r_H}^{r_H + \delta r} f dr = -f(r_H) \delta r + \mathcal{O}(\delta r^2) \quad (2.28)$$

Substituting Eqs. (2.22), (2.23) and (2.28) into Eq. (2.17), we obtain

$$\left(f(r_H) - q_I e_I - \Omega_H q_I A_\phi'^I \right) \delta r = \delta \mathcal{E} - \Omega_H^\mu \delta \mathcal{J}_\mu - (S \delta T + T \delta S). \quad (2.29)$$

This formula describes the first law of black hole thermodynamics written in terms of F . In fact, one can show that $(q_I e_I + \Omega_H q_I A_\phi^{I'}) \delta r$ is related to $\Phi \delta Q$ where $\Phi = \xi^a A_a|_{\mathcal{H}}$ is the electrostatic potential on the horizon of the hole and $Q = \int \frac{\partial \mathcal{L}}{\partial F^{ab}} \epsilon_{aba_1 \dots a_{n-2}}$ is the electric charge[26]. According to Sen's entropy function definition, the entropy function is defined by

$$\mathbf{f} = -f(r_H) + q_I e_I + \Omega_H q_I A_\phi^{I'}. \quad (2.30)$$

Finally, we come to the equation for the entropy function

$$\mathbf{f} \delta r = \delta \mathcal{E} - \Omega_H^\mu \delta \mathcal{J}_\mu - (S \delta T + T \delta S). \quad (2.31)$$

From the first law of black hole thermodynamics, we know that $\delta \mathcal{E} = T \delta S + \Omega_H^\mu \delta \mathcal{J}_\mu$, and

$$\mathbf{f} \delta r = S \delta T. \quad (2.32)$$

It becomes clear that the entropy function is closely related to the free energy via

$$\mathbf{f} = e_I q_I + \Omega_H q_I A_\phi^{I'} - \frac{\partial F}{\partial r} |_{r_H}. \quad (2.33)$$

Note that the discussions on the relations between Wald entropy formula and the entropy function method in Ref.[25] only works for spacetimes with vanishing "canonical energy" i.e. $\mathcal{E} = 0$ (for example pure de Sitter space or pure anti-de Sitter space), because the boundary condition at infinity is not included in their discussions. We also emphasize here that Eq.(2.31) describes the relations between the black hole thermodynamics and the entropy function in an asymptotically flat space. When we extend our discussions to asymptotically anti-de Sitter (AdS) or de Sitter space, we need impose corresponding boundary conditions because the mass definition in asymptotically AdS spaces is different from that in asymptotically flat spaces.

In the zero temperature limit, our results agree with the results obtained in Ref.[18, 19]. We know that $F = T I_E$, so Eq.(2.33) becomes

$$\mathbf{f} = e_I q_I + \Omega_H q_I A_\phi^{I'} - T I_E' - T' I_E. \quad (2.34)$$

When $T \rightarrow 0$, we find that

$$\mathbf{f}_{ex} = e_I q_I + \Omega_H q_I A_\phi^{I'} - T' I_E = S T'. \quad (2.35)$$

We can redefine $\tilde{e}_I = e_I / T'$, $\tilde{\mathbf{f}} = \mathbf{f}_{ex} / T'$ and $\tilde{A}_\phi^I = A_\phi^{I'} / T'$, and obtain

$$\tilde{\mathbf{f}} = \tilde{e}_I q_I + \Omega_H q_I \tilde{A}_\phi^I - I_E = S. \quad (2.36)$$

The above equation was first obtained in Ref.[18, 19]. Comparing the thermodynamic result Eq.(2.36) with the Sen's entropy function, i.e.

$$S = 2\pi(e_i q_i - f_{sen}). \quad (2.37)$$

We have (see also [18, 19])

$$\tilde{e}_I = 2\pi e_i, \quad q_I = q_i, \quad I_E = 2\pi f_{sen}. \quad (2.38)$$

We emphasize that I_E in Eq.(2.36) is defined in the zero temperature limit, in particular, the free energy is also evaluated in the zero temperature limit. In this sense, the Euclidean action in Eq.(2.36) is quite different from Euclidean action for the non-extremal black holes [33].

3. Near horizon geometry and higher derivative curvature terms

Now let us interpret Sen's entropy function method from Eq.(2.31). Usually according to Sen's entropy function method, if we can deform the near horizon geometry of extremal black holes into $AdS_2 \times S^{n-2}$, the calculation of Wald's entropy formula $S_{BH} = -2\pi \int d^{n-2}x \sqrt{h} \frac{\partial \mathcal{L}}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd}$ becomes very simple. Remarkably, even without knowing the solutions in higher derivative gravity theory, one can obtain the higher order corrections to entropy of extremal black holes (such as extremal 3- and 4-charge black holes in string theory [27, 28]). Here we explain why Eq.(2.2) makes sense in obtaining higher derivative entropy corrections.

We give a demonstration for AdS spaces together with curvature squared corrections. The general action of n -dimensional R^2 -gravity with cosmological constant and matter. The action is given by

$$I = \int d^n x \sqrt{-g} \left\{ \frac{1}{16\pi G_n} R - \Lambda + a R^2 + b R_{\mu\nu} R^{\mu\nu} + c R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \mathcal{L}_m \right\}, \quad (3.1)$$

where a, b, c are arbitrary small coefficients derived in string theory, \mathcal{L}_m is the lagrangian for the matter fields, and the negative (positive) constant Λ creates an AdS (dS) space with radius $l^2 = -\frac{(n-1)(n-2)}{16\pi G_n \Lambda}$. By the variation over the metric tensor $g_{\mu\nu}$, we obtain the equation of motion, i.e. the Einstein equation with R^2 corrections

$$\begin{aligned} \frac{1}{8\pi G_n} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 8\pi G_n g_{\mu\nu} \Lambda \right) &= T_{\mu\nu}^{matter} + a \left(\frac{1}{2} g_{\mu\nu} R^2 - 2R R_{\mu\nu} + 2\nabla_\mu \nabla_\nu R - 2g_{\mu\nu} \square R \right) \\ &+ b \left(\frac{1}{2} g_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma} + 2\nabla_\mu \nabla_\alpha R_\mu^\alpha - \square R_{\mu\nu} - g_{\mu\nu} \square R / 2 - 2R_\mu^\alpha R_{\alpha\nu} \right) \\ &+ c \left(\frac{1}{2} g_{\mu\nu} R_{\alpha\beta\rho\sigma} R^{\alpha\beta\rho\sigma} - 2R_{\mu\alpha\rho\sigma} R_\nu^{\alpha\rho\sigma} - 4\square R_{\mu\nu} + 2\nabla_\mu \nabla_\nu R + 4R_\mu^\alpha R_{\alpha\nu} - 4R^{\alpha\beta} R_{\mu\alpha\nu\beta} \right) \end{aligned} \quad (3.2)$$

Since the near horizon geometry of extremal black holes always has the geometry of $AdS_2 \times S^{n-2}$, we may consider a pure AdS space in the Poincare coordinate, where the AdS metric is considered as the near horizon limit of an extremal black hole and temperature of the AdS space is therefore zero. The unperturbed AdS metric reads

$$ds^2 = \frac{1}{z^2 l^2} (-dt^2 + d\vec{x}^2) + \frac{l^2}{z^2} dz^2, \quad (3.3)$$

where $z = 1/r$ with respect to (2.2). We choose the metric (3.3) which is the Poincare coordinate that covers part of the manifold, because the near horizon geometry of extremal black holes is always $AdS_2 \times S^{n-2}$. Following Ref.[29], we can calculate the curvature square

corrections to metric (3.3) straightforward. The perturbed metric turns out to be

$$ds^2 = \frac{1}{z^2 l^2} \left[- \left(1 + \frac{16(n-4)\pi G_n}{(n-2)l^2} ((n-1)na + (n-1)b + 2c) \right) dt^2 + d\vec{x}^2 \right] + \frac{l^2}{z^2 \left(1 + \frac{16(n-4)\pi G_n}{(n-2)l^2} ((n-1)na + (n-1)b + 2c) \right)} dz^2. \quad (3.4)$$

From (3.4), one can find that, at least to the first order of a , b and c , the R^2 corrections have no effect on the geometry of AdS space except the AdS scale. Therefore it is safe to compute the higher curvature corrections to extremal black hole entropy by using the unperturbed metric, i.e. the near horizon geometry (2.2). Actually, this is not restrict to extremal black holes with near horizon geometry $AdS_2 \times S^{n-2}$. We found that for pure de Sitter space the entropy with higher derivative corrections can also be obtained without knowing the corrected metric[30].

4. Entropy for non-extremal $D1D5$ -branes with R^4 corrections

The entropy function formalism works well for extremal black holes, since the near horizon geometry of extremal (i.e. BPS) black hole has the exact geometry of $AdS_2 \times S^{n-2}$. To extend the entropy function formalism to non-extremal black holes, we will move away from the extremality slightly. We assume the temperature of non-extremal black holes $T_H \ll 1$ and then the entropy $S = -T \frac{\partial I_E}{\partial T} - I_E \approx -I_E$. From Eq.(2.34) and Eq.(2.35), we have

$$TS' + T'S \approx \mathbf{f}_{ex}. \quad (4.1)$$

Therefore, once we know the entropy function for extremal black holes, we can obtain the non-extremal black hole entropy by considering small non-extremality. To make sure the above proposal works well for non-extremal black hole when higher derivatives corrections are included, we first make a doubt check on non-extremal $D1D5$ -branes, which was recently calculated in Ref.[31].

4.1 Entropy function method

The type II supergravity action in string frame read as

$$S_{II} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{(3)}^2 \right) - \frac{1}{2} \sum_n \frac{1}{n!} F_n^2 \right\}, \quad (4.2)$$

where ϕ is the dilaton, $H_{(3)}$ is NS-NS 3-form field strength, and $F_{(n)}$ is the electric R-R n -form field strength where $n = 1, 3, 5$ for type IIB supergravity and $n = 2, 4$ for type IIA supergravity. The near horizon geometry of non-extremal $D1d5$ -branes, which is given by the following line-elements

$$ds^2 = \frac{r^2}{\sqrt{Q_1 Q_5}} \left[- \left(1 - \frac{r_0^2}{r^2} \right) dt^2 + dy^2 \right] + \frac{\sqrt{Q_1 Q_5}}{r^2} \left(1 - \frac{r_0^2}{r^2} \right)^{-1} dr^2 + \sqrt{Q_1 Q_5} d\Omega_3^2 + \sqrt{\frac{Q_1}{Q_5}} \sum_{i=1}^4 dz_i^2$$

$$e^{-2\phi} = \frac{Q_5}{Q_1}, \quad F_{rty} = 2\frac{r}{Q_1}, \quad F_{rtyz_1 \dots z_4} = 2\frac{r}{Q_5} \quad (4.3)$$

We can see in the following that even we start from extremal $D1D5$ systems, the non-extremal $D1D5$ -branes entropy can be obtained by using (4.1). The extremal metric $D1D5$ -branes systems reads

$$ds^2 = \frac{r^2}{\sqrt{Q_1 Q_5}} [-dt^2 + dy^2] + \frac{\sqrt{Q_1 Q_5}}{r^2} dr^2 + \sqrt{Q_1 Q_5} d\Omega_3^2 + \sqrt{\frac{Q_1}{Q_5}} \sum_{i=1}^4 dz_i^2 \quad (4.4)$$

In order to apply the entropy function formalism to the extremal $D1D5$ -branes, one should deform the near horizon geometry to the most general form which is the product of the AdS-Schwarzschild and $S^3 \times T^4$ space,

$$ds^2 = v_1 \left\{ \frac{r^2}{\sqrt{Q_1 Q_5}} [-(dt^2 + dy^2)] + \frac{\sqrt{Q_1 Q_5}}{r^2} dr^2 \right\} + v_2 \left\{ \sqrt{Q_1 Q_5} d\Omega_3^2 + \sqrt{\frac{Q_1}{Q_5}} \sum_{i=1}^4 dz_i^2 \right\}$$

$$e^{-2\phi} = \frac{Q_5}{Q_1} u, \quad F_{rty} = 2 \frac{r v_1^{\frac{3}{2}}}{Q_1 v_2^{\frac{7}{2}}} \equiv e_1, \quad F_{rtyz_1 \dots z_4} = 2 \frac{r}{Q_5} v_1^{\frac{3}{2}} v_2^{\frac{1}{2}} \equiv e_2, \quad (4.5)$$

where v_1, v_2, u are assumed to be constants. The function f is defined to be the integral of the Lagrangian density over the horizon, so from (4.2) and (4.5) we can write

$$\begin{aligned} f(v_1, v_2, u, e_1, e_2, r) &\equiv \frac{1}{16\pi G_{10}} \int dx^H \sqrt{-g} \mathcal{L} \\ &= \frac{V_1 V_3 V_4 r}{16\pi G_{10}} Q_1^{3/2} Q_5^{-1/2} v_1^{3/2} v_2^{7/2} \\ &\quad \times \left(\frac{6u Q_5^{\frac{1}{2}} (v_1 - v_2)}{Q_1^{\frac{3}{2}} v_1 v_2} + \frac{Q_1^{\frac{1}{2}} Q_5^{\frac{1}{2}}}{2 v_1^3 r^2} e_1^2 + \frac{Q_5^{\frac{5}{2}}}{2 Q_1^{\frac{3}{2}} v_1^3 v_2^4 r^2} e_2^2 \right), \end{aligned} \quad (4.6)$$

where V_1 is the volume of S^1 , V_3 is the volume of the 3-sphere with unit radius, and V_4 is the T^4 volume. The entropy function is then written as

$$\mathbf{f}_{ex}(v_1, v_2, u) \equiv \left(e_i \frac{\partial f}{\partial e_i} - f \right) = \frac{V_1 V_3 V_4}{16\pi G_{10}} v_1^{3/2} v_2^{7/2} r \left(\frac{6u(v_2 - v_1)}{v_1 v_2} + \frac{2}{v_2^7} + \frac{2}{v_2^3} \right) \quad (4.7)$$

Now, we use the entropy function equation $TS' + T'S \approx \mathbf{f}_{ex}(v_1, v_2, u)$, where the surface gravity is given by $\kappa = \sqrt{g^{rr}} \frac{d}{dr} \sqrt{-g_{tt}} = \frac{r}{\sqrt{Q_1 Q_5}}$. Solving the entropy function equation, we find that

$$S = \frac{V_1 V_3 V_4}{16G_{10}} \sqrt{Q_1 Q_5} v_1^{3/2} v_2^{7/2} r \left(\frac{6u(v_2 - v_1)}{v_1 v_2} + \frac{2}{v_2^7} + \frac{2}{v_2^3} \right). \quad (4.8)$$

Extremizing S with respect to v_i and u_i , we obtain

$$\frac{\partial S}{\partial v_1} = 0, \quad \frac{\partial S}{\partial v_2} = 0, \quad \frac{\partial S}{\partial u} = 0, \quad (4.9)$$

with the following solutions

$$v_1 = v_2 = 1, u = 1. \quad (4.10)$$

Substituting the above values back to Eq.(4.8), we finally obtain the entropy for non-extremal $D1D5$ branes

$$S_{BH}|_{horizon} = \frac{V_1 V_3 V_4 r_0 \sqrt{Q_1 Q_5}}{4G_{10}}, \quad (4.11)$$

where r_0 is the event horizon radius. The result agrees with that of Ref. [31], but the method here is rather simpler.

We can see in the below that even when the higher derivative terms are included, the entropy function equation still works well. When the next leading order Lagrangian for type II theory is included, the action becomes [32]

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ \mathcal{L}^{tree} + e^{-2\phi} (\gamma W) \right\}, \quad (4.12)$$

where $\gamma = \frac{1}{8}\zeta(3)(\alpha')^3$ and W can be written in terms of the Weyl tensors

$$W = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk}. \quad (4.13)$$

For the metric configuration (4.5), the contribution of the above higher derivative terms to the entropy function

$$\Delta \mathbf{f}(v_1, v_2, u) = -\gamma u \frac{V_1 V_3 V_4 \sqrt{Q_1 Q_5}}{16\pi G_{10}} v_1^{\frac{3}{2}} v_2^{\frac{7}{2}} r \left[\frac{3(35v_2^4 - 20v_1^3 v_2 + 18v_1^2 v_2^2 - 20v_1 v_2^3 + 35v_1^4)}{32Q_1^2 Q_5^2 v_1^4 v_2^4} \right] \quad (4.14)$$

Substituting (4.7) and (4.14) back into the entropy function equation $T'S' + T'S \approx \mathbf{f}_{ex}(v_1, v_2, u) + \Delta \mathbf{f}(v_1, v_2, u)$, we find that

$$S|_{r=r_0} = \frac{V_1 V_3 V_4}{16G_{10}} \sqrt{Q_1 Q_5} v_1^{3/2} v_2^{7/2} r_0 \left(\frac{6u(v_2 - v_1)}{v_1 v_2} + \frac{2}{v_2^7} + \frac{2}{v_2^3} - \gamma \sqrt{Q_1 Q_5} \frac{3(35v_2^4 - 20v_1^3 v_2 + 18v_1^2 v_2^2 - 20v_1 v_2^3 + 35v_1^4)}{32Q_1^2 Q_5^2 v_1^4 v_2^4} \right). \quad (4.15)$$

The final result can be obtained by extremizing \mathcal{E} with respect to v_1 , v_2 and u , that is to say,

$$\frac{\partial S}{\partial v_1} = 0, \quad \frac{\partial S}{\partial v_2} = 0, \quad \frac{\partial S}{\partial u} = 0. \quad (4.16)$$

The solutions yield

$$v_1 = 1 - \gamma \frac{51}{32(Q_1 Q_5)^{\frac{3}{2}}}, \quad v_2 = 1 - \gamma \frac{27}{32(Q_1 Q_5)^{\frac{3}{2}}}, \quad u = 1 + \gamma \frac{33}{8(Q_1 Q_5)^{\frac{3}{2}}}. \quad (4.17)$$

We finally obtain the entropy of non-extremal $D1D5$ -branes under higher derivative corrections

$$S_{BH} = \frac{V_1 V_3 V_4 r_0 \sqrt{Q_1 Q_5}}{4G_{10}} \left[1 - \gamma \frac{9}{8(Q_1 Q_5)^{3/2}} + O(\gamma^2) \right]. \quad (4.18)$$

Again, we reproduce the result of Ref.[31]. Now it is safe to extend this method to more general conditions.

4.2 Wald's formula for the R^4 term

We can also calculate the R^4 correction to the entropy of $D1D5$ -branes by using the Wald expression, which is given by

$$S_{BH} = -2\pi \int_{\mathcal{H}} d^3x \sqrt{h} \frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma}, \quad (4.19)$$

where $\epsilon_{\mu\nu}$ is the binormal to the bifurcation surface, and $\epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$. The Wald entropy for AdS-Schwarzschild has been calculated in [33] and we will follow the method in [33] to calculate the entropy for $D1D5$ systems. One can choose $\epsilon_{\mu\nu} = \xi_\mu \eta_\nu - \xi_\nu \eta_\mu$, where $\xi = \frac{\partial}{\partial t}$ is the Killing vector field, and $\eta = -g_{tt}^{-1} \frac{\partial}{\partial t} - \frac{\partial}{\partial r}$ is the null vectors normal to the bifurcation Killing horizon. Now, keeping in mind that $e^{-2\phi} = \frac{Q_5}{Q_1}$, we can rewrite the R^4 corrections to the Lagrangian,

$$\Delta L = \frac{\gamma Q_5}{16\pi G_{10} Q_1} W \quad (4.20)$$

Therefore, the corrections to the entropy is given by

$$\Delta S_{BH} = -\frac{\gamma Q_5}{8G_{10} Q_1} \int_{\mathcal{H}} dx^H \sqrt{h} \frac{\partial W}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} = -\frac{\gamma Q_5}{8G_{10} Q_1} \int_{\mathcal{H}} dx^H \sqrt{h} \frac{\partial W}{\partial C_{abcd}} \frac{\partial C_{abcd}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \quad (4.21)$$

Using the unperturbed metric Eq.(4.3), one can obtain

$$\frac{\partial W}{\partial C_{abcd}} \frac{\partial C_{abcd}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} = \frac{9}{4} \frac{1}{(Q_1 Q_5)^{\frac{3}{2}}}. \quad (4.22)$$

Note that the integral of Wald entropy is defined over the horizon $H = S^1 \times S^3 \times T^4$ and $\sqrt{h} = Q_1^{\frac{3}{2}} Q_5^{-\frac{1}{2}} r_0$. Finally, we obtain

$$\Delta S_{BH} = -\gamma \frac{9V_1 V_3 V_4 r_0}{32(Q_1 Q_5)} \quad (4.23)$$

The total entropy becomes

$$S_{BH} = \frac{V_1 V_3 V_4 r_0 \sqrt{Q_1 Q_5}}{4G_{10}} \left(1 - \gamma \frac{9}{8(Q_1 Q_5)^{\frac{3}{2}}} \right), \quad (4.24)$$

which agrees with Eq.(4.18).

5. Entropy of near-extremal $D1D5p$ black holes

In this section, we extend the method of the modified entropy function to calculate near-extremal 3-charge black holes. In the following of our calculation, we assume that the 3-charge black holes is near extremal with small temperature so that we can use the extremal black hole entropy function and Eq.(4.1) to compute the entropy of near-extremal $D1D5p$ black holes.

5.1 10-dimensional case

The entropy of extremal $D1D5p$ black holes in the presence of higher derivative corrections can be easily computed using Sen's entropy function [28]. By releasing the extremal constraint slightly, we hope to compute the non-extremal 3-charge black hole approximately. The near horizon geometry of $D1D5p$ metric is $M_3 \times S^3 \times T^4$ where M_3 is the deformed AdS_3 geometry.

The non-extremal black hole metric in $n = 10$ is given as follows in string frame

$$\begin{aligned} ds_{10}^2 &= (f_1(r)f_5(r))^{-\frac{1}{2}} [-dt^2 + dz^2 + K(r)(\cosh \sigma dt + \sinh \sigma dz)^2] \\ &\quad + (f_1(r)/f_5(r))^{\frac{1}{2}} dx_{\parallel}^2 + f_1(r)^{\frac{1}{2}} f_5(r)^{\frac{1}{2}} \left[\frac{dr^2}{1-K(r)} + r^2 d\Omega_3^2 \right], \\ e^{-2\phi} &= \frac{f_5}{f_1}, \end{aligned} \quad (5.1)$$

where

$$K(r) = \frac{r_0^2}{r^2}, \quad f_1(r) \equiv 1 + \frac{Q_1}{r^2} = 1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}, \quad f_5(r) \equiv 1 + \frac{Q_5}{r^2} = 1 + \frac{r_0^2 \sinh^2 \chi}{r^2}, \quad (5.2)$$

The conserved charges carried by this hole are

$$N_1 = \frac{V r_0^2 \sinh 2\alpha}{2g_s \alpha'^3}, \quad N_5 = \frac{r_0^2 \sinh 2\chi}{2g_s \alpha'}, \quad N_p = \frac{R_z^2 V r_0^2 \sinh 2\sigma}{2g_s^2 \alpha'^4}, \quad (5.3)$$

where $V = R_5 R_6 R_7 R_8$, $R_i (i = 5, 6, 7, 8)$ denote the radii of the four coordinates in x_{\parallel} , and R_z is the radius of the compact dimension z , along which there is a momentum P . The extremal limit can be obtained by

$$r_0 \rightarrow 0, \quad \alpha \rightarrow \infty, \quad \chi \rightarrow \infty, \quad \sigma \rightarrow \infty, \quad (5.4)$$

while holding Q_1, Q_5 , and $Q_p = r_0^2 \sinh^2 \sigma$ fixed. The near horizon geometry of extremal $D1D5p$ system has the following form

$$\begin{aligned} ds^2 &= v_1 \left(\frac{Q_p - r^2}{\sqrt{Q_1 Q_5}} dt^2 + \sqrt{Q_1 Q_5} \frac{dr^2}{r^2} + \frac{Q_p + r^2}{\sqrt{Q_1 Q_5}} dz^2 - 2 \frac{Q_p}{\sqrt{Q_1 Q_5}} dt dz \right) \\ &\quad + v_2 \left(\sqrt{Q_1 Q_5} d\Omega_3^2 + \sqrt{\frac{Q_1}{Q_5}} dx_i^2 \right). \end{aligned} \quad (5.5)$$

When $Q_p = 0$, the geometry is $AdS_3 \times S^3 \times T^4$. The entropy function for extremal $D1D5p$ black holes can be obtained from (4.2) and (5.5) (see also [28]),

$$\mathbf{f}_{ex} = \frac{4\pi^3 R V T^4}{16\pi G_{10}} r Q_1^{\frac{3}{2}} Q_5^{-\frac{1}{2}} v_1^{\frac{3}{2}} v_2^{\frac{7}{2}} \left[\frac{6u_s(v_2 - v_1)}{(Q_1 Q_5)^{\frac{1}{2}} v_1 v_2} + \frac{Q_5^{\frac{1}{2}}}{Q_1^{\frac{3}{2}}} \left(\frac{2}{v_2^3} + \frac{2}{v_2^7} \right) \right], \quad (5.6)$$

where the value of v_1, v_2, u_s can be obtained by extremizing the entropy function

$$v_1 = v_2 = u_s = 1. \quad (5.7)$$

Substituting the above formula into the equation

$$TS' + T'S \approx \mathbf{f}_{ex}, \quad (5.8)$$

where $T = \frac{1}{2\pi r_0 \cosh \alpha \cosh \chi \cosh \sigma}$, again we obtain

$$S_{BH} = 2\pi\mathcal{E} = \frac{2\pi RV r_0^3}{g_s^2 \alpha'^4} \cosh \alpha \cosh \chi \cosh \sigma. \quad (5.9)$$

Actually, the entropy functions for extremal $D1D5p$ black holes at $n = 5$ and $n = 10$ are identified with each other. So, we reproduce the entropy for non-extremal $D1D5p$ black holes.

5.2 R^4 corrections to non-extremal 10-dimensional $D1D5p$ black hole entropy

The R^4 corrections to extremal 3-charge black hole entropy was obtained in [28]. In this subsection, we will extend the previous results to non-extremal case. Usually there are several forms of higher order of corrections in type II superstring theory. we can consider the higher derivative corrections in [32]

$$L_{corr} = \gamma e^{-2\phi} (L_1 - 2L_2 + \lambda L_3),$$

$$L_1 = R^{hmnk} R_{pmnq} R_h{}^{rsp} R_{rsk}^q + \frac{1}{2} R^{hkmn} R_{pqmn} R_h{}^{rsp} R_{rsk}^q, \quad (5.10)$$

$$L_2 = R^{hk} (\frac{1}{2} R_{hnpk} R^{msqn} R_{msq}{}^p + \frac{1}{4} R_{hpmn} R_k{}^{pqs} R_{qs}{}^{mn} + R_{hmn p} R_{kqs}{}^p R^{nqsm}),$$

$$L_3 = R(\frac{1}{4} R_{hpmn} R^{hpqs} R_{qs}{}^{mn} + R_{hmn p} R_{qs}{}^p R^{nqsm}),$$

where $\gamma = \frac{1}{8}\zeta(3)\alpha'^3$ and λ is a parameter which signifies the ambiguity in the field redefinitions of the metric. Now the corrected entropy function becomes

$$\begin{aligned} \mathbf{f}_{ex} + \Delta \mathbf{f} = & \frac{4\pi^3 RV T^4}{16\pi G_{10}} r Q_1^{\frac{3}{2}} Q_5^{-\frac{1}{2}} v_1^{\frac{3}{2}} v_2^{\frac{7}{2}} \left[\frac{6u_s(v_2 - v_1)}{(Q_1 Q_5)^{\frac{1}{2}} v_1 v_2} + \frac{Q_5^{\frac{1}{2}}}{Q_1^{\frac{3}{2}}} \left(\frac{2}{v_2^3} + \frac{2}{v_2^7} \right) \right. \\ & \left. - 6\gamma u_s \frac{7v_1^4 + 7v_2^4 + 6\lambda Q_5^{\frac{1}{2}}(v_1^4 - v_2 v_1^3 + v_2^4 - v_1 v_2^3)}{v_1^4 v_2^4 Q_1^2 Q_5^2} \right], \end{aligned} \quad (5.11)$$

The solutions to the equations of motion of moduli by extremizing the corrected entropy function are found to be

$$v_1 = 1 + \frac{63\gamma}{11(Q_1 Q_5)^{3/2}}, \quad v_2 = 1 - \frac{91\gamma}{11(Q_1 Q_5)^{3/2}}, \quad u_s = 1 + \frac{567\gamma}{11(Q_1 Q_5)^{3/2}}. \quad (5.12)$$

Then we obtain the corrected entropy function

$$\mathbf{f}_{ex} + \Delta \mathbf{f} = \frac{4\pi^3 RV T^4}{16\pi G_{10}} r \left(1 - \frac{84\gamma}{(Q_1 Q_5)^{3/2}} \right) \quad (5.13)$$

Then the entropy is given by

$$S_{BH} = \frac{2\pi RV r_0^3}{g_s^2 \alpha'^4} \cosh \alpha \cosh \chi \cosh \sigma \left(1 - \frac{84\gamma}{(Q_1 Q_5)^{3/2}} \right). \quad (5.14)$$

When $\kappa \rightarrow 0$, we return to the extremal case with R^4 corrections.

6. Conclusions

In summary, we have proved that the entropy function is related to the free energy via $\mathbf{f} = e_{IQI} + \Omega_{HQI} A_\phi^{I'} - \frac{\partial F}{\partial r} |_{r_H}$. In this sense the entropy function is nothing special but a transformed version of black hole thermodynamics. We have found that the secret of the entropy formalism relies on the near horizon geometry of the metric. While the near horizon geometry of a extremal black hole might have $SO(2, 1) \times SO(D-1)$ isometry (for spherically symmetric black holes) or $SO(2, 1) \times U(1)$ isometry (for rotating holes) in higher derivative gravity theory is not generally proved, we have pointed out that the higher derivative terms do not change the geometry of AdS space in Poincare coordinate. We have also proposed that when we slightly move the black hole geometry from extremality the extremal entropy function is still valid. In this case, the near-extremal black hole entropy and the extremal black hole entropy function approximately satisfy a equation $TS' + T'S \approx \mathbf{f}_{ex}$. One can solve this equation to obtain the near-extremal black hole entropy. As concrete examples, we calculate the entropy of non-extremal $D1D5$ - and $D1D5p$ -branes in the presence of higher derivative corrections. Although the above method is not rigorously established, our computations agree with the results obtained in previous literature.

Acknowledgements

The authors would like to thank G. W. Kang, S. P. Kim, M. I. Park, P. Zhang and S. Q. Wu for their helpful comments at different stages of this work.

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